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HOW MUCH DO FOOD PRICES MATTER FOR MEN'S AND WOMEN'S BODY WEIGHT?

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Abstract

This article contributes to the literature that studies the impact of food prices on food choices and obesity rates. While it is now well established that daily calories consumption in excess of dietary guidelines and the switch toward more sedentary lifestyles are key factors for the rise in obesity prevalence of American men and women after 1970, there is much less consensus about what caused eating habits of men and women to change over time. We analyze the impact of food prices on body weight in a dynamic setting where men and women have different preferences and choose between J food groups. We derive an analytical expression linking preference parameters, including the elasticity of substitution, to empirical estimates of price and cross-price elasticity of demand for the J food groups. One key takeaway from the calibration is that there is substantial preference heterogeneity between men and women. We find that most food groups are substitutes and thus an increase in food prices, perhaps due to a sin tax, does not always lead to body weight losses.
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JEL Classification: *I10, D91*

1. Introduction

The economic costs and consequences of obesity are now well established in the health economics literature (e.g, Cawley (2015)). We also know that daily calories consumption in excess of dietary guidelines and the switch toward more sedentary lifestyles are key factors responsible for the rise in obesity prevalence after 1970 (Cutler et al., 2003). What there is less concensus about however is what caused eating habits of men and women to change over time (Cawley, 2011)?

Empirical estimates of the price and cross-price elasticity for the demand of food items are usually small and thus changes in food prices tend to have a negligible effect on body weight (Cawley et al., 2019; Fletcher et al., 2015). In addition, the theoretical assumptions underpinning empirical work on food choices and body weight are not always fully worked out. For example, internatilities, the future costs of excess consumption that are ignored at the point of consumption, are often mentioned as important determinants of food choices but few papers directly model food choices as intertemporal decisions (see Lakdawalla and Philipson (2009) and Buttet and Dolar (2015b)).

Following insights by Lakdawalla et al. (2005) and Lakdawalla and Philipson (2009),

we propose a quantitative recursive model where food consumption choices can be formulated as a dynamic program and where body weight, the state variable, also enters the utility function. The assumptions used in these two papers are interesting because state variables do not usually enter agents' utility in dynamic economic models. For example, in the one-sector growth model of macroeconomics, market goods are produced using physical or human capital (the state variables) as inputs. However, only the stream of market good consumption enters agent's utility, not physical or human capital (Ljungqvist & Sargent, 2012).

In the field of obesity economics, however, there are good reasons to believe that weight in the utility function makes sense. First, weight is a proxy for health. Today, Americans are heavier than what the medical field recommends and the obesity epidemic is associated with many of the leading causes of preventable death such as heart disease, stroke, type-II diabetes and certain types of cancer (National Institute of Health, 2005). Second, people care about the way they look and being too skinny or too fat can affect people's self-esteem above and beyond medical considerations.

We extend Lakdawalla et al. (2005) and Lakdawalla and Philipson (2009)'s framework by nesting J types of food with a constant elasticity of substitution function. Since the obesity economics literature emphasizes how important substitution between food items is when it comes to body weight analysis, we believe that studying the properties of a theoretical framework where agents choose among several food choices is a nice addition to the literature.

In the spirit of the research program which uses calibration, our model is very stylized with only a small number of preference parameters that needs to be calibrated (Buttet & Dolar 2015a; Kalamov, 2020). We use data moments for total calories, food shares, and weight for men and women from the 1971 National Health and Nutrition Examination Survey (NHANES I) to calibrate preferences parameters where we aggregate the food groups contained in NHANES I into five food composites: fruits and vegetables, dairy, meat, grains, and fat and sugar.

We also derive an analytical expression linking preference parameters, including the elasticity of substitution, to empirical estimates of price and cross-price elasticity of demand for our five food categories. Our paper thus brings together two strands of the literature by showing how estimates from the micro-literature can be used to calibrate dynamic models of food choices and body weight. One interesting result from the calibration is that the elasticity of substitution between food types is negative for both men and women. The main takeaway from the calibration, however, is that there is substantial preference heterogeneity between men and women. For example, utility losses stemming from weight gain are 20 percent greater for women compared to men.

Finally, we derive an expression for the price and cross-price elasticity for food demand for men and women and circle back to the research question posed earlier: how do substitution between different food group affect food choices and body weight? As expected the demand for any food category is downward sloping. However, we also find that most food groups are substitutes and thus an increase in food prices, perhaps through a sin tax, does not always lead to body weight losses (Schroeter et al., 2008). Changes in food prices affect men's and women's body weight differently due to gender preference heterogeneity. Interestingly, our model predicts that a 1 percent increase in the price of fat and sugar category leads to a body weight reduction of 0.06 percent for men and 0.08 percent for women.

Our analysis highlights the importance of carefully modeling how body weight changes affect utility for men and women. We realize that the assumption of perfect rationality might be too strong when it comes to food choices. People with self-control problems or time-inconsistent preferences would find it optimal to discount future health costs and choose

immediate gratification from food consumption. Adapting the bounded rational- ity modeling strategies in Gruber and Koszegi (2004), Gruber and Koszegi (2001), or O'Donoghue and Rabin (2006) to study the impact of declining food prices on weight would be a valuable contribution to the economics of obesity literature.

The remainder of the paper is organized as follows. In the next Section, we present a dynamic model of food choices and body weight and formulate food choices as a dynamic program. In Section 3, we discuss parameters' identification and functional forms choices. In Section 4, we use data moments on calorie intake, relative food prices, and dietary guidelines to calibrate preference parameters for men and women. In Section 5, we present an expression for the price and cross-price elasticity for the demand for food items. We use these estimates to analyze the impact of changes in food prices on body weight for men and women. Finally, we offer concluding remarks in Section 6.

2. A Recursive Model of Eating Decisions and Weight

Here we present an infinite horizon dynamic model of food choices and weight. Time is discrete, t=1,2,... We denote by c_t and c_{ft} agent's consumption of the market good and food consumption in period t, respectively. There are J types of food with J>1. We denote by $f_t = (f_{1t},\ldots,f_{Jt})$ the vector of calorie intake of type-j food in period t and by $\theta_t = (\theta_{1t,\ldots,} \theta_{Jt})$ the vector of food shares with $\theta_{jt} = f_{jt} / (f_{1t}+\ldots+ f_{Jt})$ for all j=1,…,J.

We denote by $\Phi : R^J \rightarrow R_+$ the function that aggregate type-j food calorie intake into food consumption c_{ft}:

$$c_{ft} = \Phi(f_t) \tag{1}$$

We assume that Φ is increasing, concave, and continuously differentiable. We also assume that Φ is homogenous of degree one so that for any real number λ and vector u we have $\Phi(\lambda u) = \lambda\Phi(u)$.

We denote by $U(c_t,c_{ft},W)$ the period-t utility function which is assumed to be continuously differentiable, concave and increasing in market good and food consumption, c_t and c_{ft} respectively. In addition, there exists a best weight $W^{\wedge}>0$ such that $U_3(W^{\wedge}) =0$ where U_3 denotes the partial derivative of U with respect to W. Households choose sequences of market and food consumption $\{ c_t , f_t \}t\geq 1$ to maximize the discounted sum of period-t utility:

$$\sum\delta^{t-1} U(c_t,c_{ft},W) \tag{2}$$

where $\delta \in (0, 1)$ is the pure time discount factor. The budget constraint of the representative agent is given by:

$$c_t + p_t \cdot f_t = I_t \tag{3}$$

where $p_t = (p_{1t,\ldots,} p_{Jt})$ is the price per calorie vector for each type-j food, I_t denotes real income, and $p_t \cdot f_t$ is the inner product of price and calories vectors with $p_t * f_t = \sum_j p_{jt} f_{jt}$. We normalize the price of non-food to one. Finally, the inter-temporal weight law of motion links weight in the next period to current weight and calorie consumption:

$$W_{t+1}= W_t+ \zeta(1*f_t - \mu(W_t)) \tag{4}$$

Where 1=(1,…,1) is the 1-by-J row vector of ones and $1*f_t = f_{1t}+\ldots+f_{Jt.}$ equals total calories consumed in period t; $\zeta>0$ is a parameter that converts calorie consumption into weight gain and $\mu(W_t)$ is the number of calories needed to maintain a constant weight with $\mu'(W_t)>0$.

For any given sequence of prices and income, $\{p_t, I_t\}_{t\geq 1}$, and an initial weight, W_1, the representative agent chooses an optimal sequence of market and food consumption $\{c_t, f_t\}_{t\geq 1}$ to maximize the objective function in equation (2) subject to the food aggregation equation (1), the budget constraint (3), the weight law of motion (4), and non-negativity constraints for calorie and non-food consumption.

The consumer optimization problem above has a recursive structure and thus can be formulated as a dynamic programming problem where weight is the state variable (Buttet and Dolar, 2015; Lakdawalla and Philipson, 2009). We denote by V(W) the value function which is determined by the functional Bellman equation:

$$V(W) = \max \{U(c,c_f,W)+\delta V(W')\} \tag{5}$$

subject to constraints (1), (3), and (4) and where W' denotes weight in the next period. After we substitute the budget constraint and food aggregation function into the utility function, the Bellman equation becomes:

$$V(W) = \max \{U(I-pf, \Phi(f),W)+\delta V(W + \zeta(1*f - \mu(W)))\} \tag{6}$$

Assuming that the value function V is differentiable, the first-order conditions for optimality are:

$$-p_j U_{1+} \Phi_j U_{2+} \delta V'(W') \text{ for } j =1,,J \tag{7}$$

Where V' is the first-order derivative of the value function. In addition, the envelope theorem yields:

$$V'(W)=U_3 \tag{8}$$

We solve the above dynamic program when the weight is in steady state defined as $W'=W=W^*$. We denote by f^* calorie consumption in steady state. Given the weight law of motion (4), f^* is given by:

$$W^*=\mu^{-1}(1*f^*) \tag{9}$$

From first-order conditions in equation (7), we obtain for all j = 2, ..., J :

$$(p_j - p_1)U_1 = (\Phi_j - \Phi_1)U_2 \tag{10}$$

In addition, substituting the envelope condition (8) into the first-order condition (7) yields :

$$p_1U_1 = \Phi_1U_2 + \delta\zeta U_3 \tag{11}$$

We make two important comments about modeling choices. First, note from the weight law of motion in equation (4) that it does not matter what type of food is consumed. Research by Buchholz and Schoeller (2004) supports the view that a "calorie is a calorie" regardless of macronutrient composition implying that daily caloric intake, not what people eat, determines body weight (see also Nestle (2012)). On the other hand, evidence from the medical literature increasingly suggests that the type of food people eat as well as how much they eat affect health, especially longevity. For example, Johnson et al. (2013) find that reductions in nutrient intake in the absence of malnutrition, in particular reduce intake of insulin, extends lifespan in many different species. Our theoretical framework, however, does not directly consider the impact of weight on health which explains why we choose not to model nutritional value of what people eat.

Second, as highlighted in the Introduction section, our model shares many similarities with frameworks proposed by Philipson and Posner (2003) and Lakdawalla and Philipson (2009), in particular food consumption choices are dynamic and weight appears in the utility function. There are two important differences, however. First, we do not model calories expenditures. Cutler et al. (2003) show that declines in energy expenditures in the U.S. are too small to account for the observed changes in body weight after 1965. Since we study body weight changes after 1970, not modeling energy expenditures seems reasonable enough. Second, agents can consume different food types. Since the obesity economics literature emphasizes how important substitution between food items is when it comes to body weight analysis, we believe that studying the properties of a theoretical framework where agents choose among several food choices is a nice addition to the literature.

3. Identification

3.1 Functional Forms

We choose the following functional form for utility (see Lakdawalla and Philipson (2009)):

$$U(c,c_f,W) = \nu\, c_f + \frac{c}{1+\kappa\,(W-W^{\wedge})^2} \tag{12}$$

The function Φ that nests type-j food calories intake into food consumption is given

$$\Phi(\mathbf{f}) = \left(\sum_j a_j f_j^{\rho}\right)^{1/\rho} \tag{13}$$

with $0<a_j<1$ and $\sum_j a_j = 1$.

The subsistence calorie function in equation (4) is linear and given by:

$$\mu(W) = \beta_0 + \beta_1 W \tag{14}$$

with $\beta_1>0$. Given the weight law of motion in equation (4), the steady-state weight is equal to $W^* = (\sum_j f_j - \beta_0)/\beta_1$. As a result, the system of first-order equations in (10) and (11) are given by:

$$(p_j - p_1) / A = \nu \left(\sum_j a_j f_j^{\rho}\right)^{(1-\rho)/\rho} \left(a_j f_j^{\rho-1} - a_1 f_1^{\rho-1}\right) \tag{15}$$

With $A = 1+ \kappa\,((\sum_j f_j - \beta_0)/\beta_1 - W^{\wedge})^2$ and

$$p_1 / A = \nu\, a_1\, f_1^{\rho-1} \left(\sum_j a_j f_j^{\rho}\right)^{(1-\rho)/\rho} - 2\delta\zeta\kappa * (I - \sum_j p_j f_j) * ((\sum_j f_j - \beta_0)/\beta_1 - W^{\wedge}) / A^2 \tag{16}$$

3.2 Identification

Our model has J+3 parameters that needs to be calibrated. We show how to use estimates from the obesity microeconomic literature, especially about food price elasticity in Reed et al. (2005), to identify the model parameters.

Proposition1. Identification of ρ

Suppose that $(\sum_j f_j - \beta_0)/\beta_1 = W^{\wedge}$ and let ε^P denote the price and cross-price elasticity of food and $\pi_j = p_j / p_1$ the relative price of food item j. Then the parameter ρ is given by:

$$\rho = 1 - 1/((1-\pi_2) * B) \tag{17}$$

where $B = (\sum \varepsilon_{j1}^{P} \pi_j \theta_j)/(\sum \pi_j \theta_j) + \varepsilon_{11}^{P}/(\pi_2 - 1) + \varepsilon_{21}^{P}/(1/\pi_2 - 1)$

The remaining preference parameters are given by:

$$a_1 = \theta_1^{1-\rho} / \left(\sum_j \pi_j \theta_j^{1-\rho}\right) \quad \text{and} \quad a_j = a_1 \pi_k \left(\frac{\theta_k}{\theta_1}\right)^{1-\rho} \tag{18}$$

$$\nu = p_1 \theta_1^{(1-\rho)/\rho} / (a_1^{1/\rho} * \left(\sum_j \pi_j \theta_j\right)^{(1-\rho)/\rho}) \tag{19}$$

$$\kappa = - p_1 \beta_1 / (2\zeta\delta \sum_j f_j \sum \varepsilon_{j1}^{P} \theta_j) * (1 - p_1(1-\rho)*C) / (I - \sum_j p_j f_j) \tag{20}$$

where $C = (\sum \varepsilon_{j1}^{P} \pi_j \theta_j)/(\sum \pi_j \theta_j) - \varepsilon_{11}^{P}$

4. Calibration

In this section, we present the empirical components needed to calibrate the model parameters, including data on daily calorie intake of different food categories, the relative food prices of these categories, and dietary guidelines.

4.1 Total Calories, Food Shares, and Weight

Data for daily calories consumption comes from the 24–Hour Food Consumption Intake Tape of the first National Health and Nutrition Examination Survey (NHANES I) which was conducted by the National Center for Health Statistics from April 1971 through June 1974. The 24–Hour Food Consumption Intake Tape were obtained by the 24–hour recall method by which each of the 20,749 sample persons was asked to provide such information as specific food items and their quantities ingested for all regular meals, between meal foods or snacks consumed on the day, midnight to midnight, preceding the interview for each sample person interviewed. It includes foods eaten on Monday through Friday, but generally excludes foods eaten on weekends. The 24–Hour Food Consumption Intake Tape contains a separate record for each food item consumed by each examined person together with the amounts of calories, the ingestion period, the approximate time of day the food was consumed, and the food source.

Food items in the 24–Hour Food Consumption Intake Tape were originally grouped into 16 different categories: Dairy Product; Meats; Poultry; Shellfish; Fish; Eggs; Soups; Fats; Legumes/nuts; Cereals and Grain Products; Fruits and Vegetables; Sugar Products and Candies; Desserts; Condiments/Miscellaneous; Mixed Protein Dishes; Salty Snacks. We aggregate the 16 food products into five at-home food composites as follows. The fruit and vegetable composite includes fresh fruit, fresh vegetables, and processed fruit and vegetables. The dairy composite includes fluid milk, butter, cheese, and ice cream. The meat composite includes beef, pork, other meat, poultry, and fish and seafood. The grains composite includes all cereal and bakery products. Finally, the fat and sugar composite includes sugar and sweets, fats and oils, non-alcoholic beverages, eggs, and miscellaneous foods.

We select men and women age greater than 20 and analyze data for total calories consumed, weight, and food shares for men and women separately (see Table 1). An interesting finding of Table 1 is that food shares are quite similar for men and women. However, because total calories are greater for men compared to women, men's observed weight is greater compared to women's weight.

4.2 Relative Food Prices

We explain how to use household expenditures data on different food types to construct the price per calorie. As pointed out by Goldman et al. (2011) and Christian and Rashad (2009), the use of price per calorie is superior to using standard price index since the index does not take into account differential impacts on body weight of consuming various foods.

We define price per calorie for each food type, p_{jt}, with j = 1, ..., 5 by the following expression:

$$pjt = \frac{\textit{Per capita daily expenditures on food j in year t}}{\textit{Per capita daily calories produced of food j in year}} \tag{21}$$

Table 1: Total calories, food shares, and weight by gender, NHANES I 1971-75 (linearized standard deviation in parenthesis)

	Males	*Females*
Total Calories	2433 (27.4)	1538 (14.1)
Food Shares:		
Fruits and Vegetables, θ_1	0.12 (0.003)	0.14 (0.002)
Dairy, θ_2	0.12 (0.003)	0.13 (0.003)
Meats, θ_3	0.23 (0.004)	0.21 (0.004)
Grains θ_4	0.17 (0.002)	0.17 (0.002)
Fat and sugar, θ_5	0.36 (0.005)	0.35 (0.004)
Weight (in lbs):	173.1 (0.58)	144.0 (0.54)

We use multiple data sources to calculate the price per calorie. U.S. Department of Agriculture data on household expenditures shows that families spent 3 percent of their income on fruits and vegetable for the period between 1971 and 1975, while expenditure shares on dairy, meat, grains, fat and sugar was 2.7 percent, 6 percent, 2.2 percent, 3.6 percent respectively.

Second, data from the Bureau of Economic Analysis shows that real daily income per household expressed in 2009 dollars is equal to I = \$51.71. The per capita daily expenditures on food j in year t in equation (21) is obtained by multiplying expenditure shares for the five different food categories by daily real income.

Finally, U.S. Department of Agriculture data on calories produced shows that 191, 231, 537, 419, and 692 calories were produced for the period between 1971 and 1975 for our 5 food categories.

As a result, we find from equation (21) that the *price per one thousand calories* in 1972 expressed in 2009 constant dollars for the five food categories is equal to by p_{1972} = (\$8.06, 6.01, 5.80, 2.77, 2.66). The relative price of food-j which we will use heavily to calibrate model parameters in the next section are presented in Table 2.

Table 2. Relative Food Prices - 1972

Fruits and Vegetables, π_1	1
Dairy, π_2	0.74
Meats, π_3	0.72
Grains π_4	0.34
Fat and sugar, π_5	0.33

4.3 Weight Law of Motion

The weight law of motion (4) contains three parameters (ζ, β_0, β_1) that must be calibrated. The parameter ζ relates changes in body weight to total calorie consumed above and beyond what is needed to maintain a constant weight. It is well established in the nutrition literature that people gain ten pounds per year if they eat an extra one hundred calories every day above and beyond the recommended daily calorie intake (Shils et al. (1998)). Accordingly, we set $\zeta = 2.7397\ 10^{-4}$.

Second, the minimum number of calories required to maintain a constant weight is equal to $\mu(W) = \beta_0 + \beta_1$ where W denotes body weight measured in pounds. One hundred years ago, Harris and Benedict (1918) proposed an equation to estimate men's and women's basal metabolic rate (BMR) and daily kilocalorie requirements (see also Roza and Shizgal (1984)). The estimated BMR value is multiplied by a number that corresponds to the individuals' activity level and the resulting number is the recommended daily kilocalorie intake to maintain current body weight. They found that daily calorie requirements differ for men and women and the heavier an individual is the more calories need to be consumed to maintain a constant weight. We use a recent technical report on dieting and energy intake by the Food and Nutrition Board (2002, p.185) which finds that, assuming a moderate level of physical activity, men need to consume an addition 8.09 calories per day for each extra pound to maintain a constant weight, while calorie requirements for women are equal to an additional 4.76 calories per day for each extra pound. As a result, we set $\beta^m = 8.09$ and $\beta^f = 4.76$.

Third, we choose β_0 so that the steady state weight of men and women is equal to their weight in 1971 (see Table 1). Note that if individuals underreport total calories consumed daily in NHANES, then the calibrated value for β_0 is lower than what true calorie consumption would imply (Courtemanche et al., 2015; Cawley, 2004). According to equation (9), we have $\beta_{0,m} = 2433 - 8.09 * 173.1 = 1032.6$ and $\beta_{0,f} = 1538 - 4.76 * 144 = 844$.

4.4 Preference Parameters

We use equations (17) to (20) as well as data on calorie intake and relative food prices in Tables 1 and 2 respectively to calculate the 8 preference parameters (ρ, $\{a_j\}_{1,\ldots,5}$, ν, κ). We calibrate parameters for men and women separately and summarize results in Table 3. Note from equation (17) that the parameter ρ is identified by relative prices, calorie shares, and price and cross-price elasticity.

We use data from Reed et al. (2005) for price and cross-price elasticity with respect to the price of fruits and vegetables with $\varepsilon^P = (-.979, -.143, .072, .399, -.125)$. We follow the following intermediate steps to calculate ρ_m for men. According to data about food shares shown in Table 1 and relative food prices shown in Table 2, we have $\sum_j \pi_j \theta_j^m = .12 + .74 \times .12 + .72 \times .23 + .34 \times .17 + .33 \times .36 = .551$. Using elasticity data, we also have $\sum \varepsilon_{j1}^P \pi_j \theta_j^m = -.979 \times .12 - .143 \times .74 \times .12 - .072 \times .72 \times .23 + 0.399 \times .34 \times .17 - .125 \times$

$.33 \times .36 = -0.134$. As a result, the parameter ρ_m is equal to ρ_m = -.2346 (see equation (17)).

We proceed in a similar fashion to calculate ρ_f for women. According to data about food shares shown in Table 1 and relative food prices shown in Table 2, we have $\sum_j \pi_j \theta_j^f = 14 + .74 \times .13 + .72 \times .21 + .34 \times .17 + .33 \times .35 = .561$. Using elasticity data, we also have $\sum \varepsilon_{j1}^P \pi_j \theta_j^m = -.979 \times .14 - .143 \times .74 \times .13 - .072 \times .72 \times .21 + 0.399 \times .34 \times .17 - .125 \times .33 \times .35 = -0.153$. As a result, the parameter ρ_f is equal to ρ_f = -.2464 (see equation (17)).

Once the parameter ρ is calibrated we use equations (18) to (20) to calculate the remaining preference parameters where we set the discount factor to $\delta = .98^{1/365}$. Calibrated parameters values for men and women are shown in Table 3 below.

One interesting result from the calibration is that the parameter ρ is negative for both men and women. The main takeaway from the calibration however is that there is substantial preference heterogeneity between men and women. For example, the parameter κ which measures utility losses when steady-state weight deviates from best weight is 20 percent greater for women compared to men.

We do not seek to explain preference heterogeneity between gender in this paper. We note however that the larger utility costs of being obese for women are consistent with the literature on discrimination in the workforce toward obese women (Cawley, 2004).

Table 3. Calibrated Parameters by Gender

	Males	*Females*
ρ	-.2346	-.2464
a_1	.1941	.2307
a_2	.1436	.1556
a_3	.3121	.2752
a_4	.1015	.0999
a_5	.2487	.2385
v	.0227	.0234
κ	.0187	.0224

5. Discussion

We can now return to the question we posed in the Introduction section: how do substitution between different food group affect food choices and body weight? We denote by ε^P denote the J-by-J matrix of price and cross-price elasticities for type-j food. In the next proposition, we show that the elasticity matrix is given by:

Proposition 2. Price and cross-price elasticities are equal to:

$$\varepsilon^P = \Gamma^{-1} \Lambda \tag{22}$$

Using calibrated parameter values in Table 3, we calculate the price elasticity for men and women and present results in Tables 4 and 5, respectively. Our model predicts that increases in the price of any food item leads to reduction in the demand for this food item. However, we also find that most food groups are substitutes and thus an increase in food prices, perhaps through a sin tax, does not always lead to body weight losses (Schroeter et al., 2008). As in Reed et al. (2005), we find that fruits and vegetables are the most price elastic food category. Fat and sugar however is the least elastic category, not meat.

Table 4. Price and Cross-Price Elasticities Men

	Fruits and Vegetables	Dairy	Meats	Grains	Fat and sugar
Fruits and Vegetables	**-0.957**	0.011	0.026	0.082	0.177
Dairy	-0.113	**-0.773**	0.073	0.098	0.211
Meats	-0.109	0.039	**-0.732**	0.100	0.214
Grains	0.042	0.151	0.287	**-0.637**	0.364
Fat and sugar	0.050	0.157	0.299	0.177	**-0.438**

Table 5. Price and Cross-Price Elasticities Women

	Fruits and Vegetables	Dairy	Meats	Grains	Fat and sugar
Fruits and Vegetables	**-0.968**	0.014	0.027	0.082	0.172
Dairy	-0.126	**-0.761**	0.069	0.098	0.205
Meats	-0.122	0.044	**-0.728**	0.100	0.208
Grains	0.052	0.164	0.262	**-0.630**	0.352
Fat and sugar	0.062	0.171	0.273	0.176	**-0.442**

In the next Proposition, we derive a formula for the price-weight elasticity vector $\boldsymbol{\varepsilon^W}$ from our model. We present estimates for weight elasticity for men and women in Table 6.

Proposition 3. **Price and cross-price elasticities are equal to:**

$$\varepsilon^W=(1+ \beta_0/(\beta_1 W))\ \varepsilon^P\theta \tag{23}$$

Our model shows that changes in food prices affect men's and women's body weight differently. For example, an increase in the price of dairy leads to an increase in men's weight and a reduction in women's weight. Second, not all weight elasticity are negative as we expect from Tables 4 and 5. In particular our model predicts that an increase of 1 percent in the price of grains will lead to a 0.21 percent increase in weight for men and 0.22 percent for women. Interestingly, we find that a 1 percent increase in the price of fat and sugar category leads to a weight reduction of 0.06 percent for men and 0.08 percent for women.

Table 6. Price Elasticity for Men's and Women's Weight

	Males	*Females*
Fruits and Vegetables	-0.057	-0.120
Dairy	0.006	-0.030
Meats	-0.156	-0.166
Grains	0.210	0.223
Fat and Sugar	-0.064	-0.081

6. Concluding Remarks

In this paper, we presented a quantitative dynamic model of food choices and weight to analyze substitution among different food categories and the resulting impact on weight. Dynamic models have the advantage to directly model households' internalities for food choices as future utility costs and benefits are taken into account for contemporaneous food choices.

Our paper also attempts to bridge the gap between dynamic models and the applied

microeconomic literature as we showed how to use elasticity estimates to calibrate model parameters. One important takeaway from the calibration is that there is substantial preference heterogeneity for how obesity affects men and women. Finally, we derived an expression for price and cross-price elasticity food choices and weight for men and women. Consistent with previous studies, we find that a 1 percent increase in the price of fat and sugar category leads to a weight reduction of 0.06 percent for men and 0.08 percent for women (Allcott et al., 2019).

We see two important directions for future research. First, the assumption of perfect rationality might be too strong when it comes to food choices. Instead bounded rationality might be more appropriate (O'Donoghue and Rabin, 2006; Gruber and Koszegi, 2001). Second, our results show substantial heterogeneity in men's and women's preferences. Carefully developing separate dynamic theories of food choices and body weight for men and women which reflect both institutional and economic constraints might prove a worthwhile research agenda.

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Appendix

A. Proof of Proposition 1

We first show how to rewrite first-order condition (15) as a function of price and cross-price elasticity. For all j = 2,...,J, equation (15) reads:

$$\frac{p_j - p_1}{1+\kappa(\frac{\sum_s f_s - \beta_0}{\beta_1} - \bar{W})^2} = \nu(\sum_{s=1}^{s=J} a_s f_s^{\rho})^{\frac{1-\rho}{\rho}}(a_j f_j^{\rho-1} - a_1 f_1^{\rho-1}) \tag{24}$$

Take logarithm of the previous expression.

$$\ln(p_j - p_1) - \ln(1+\kappa(\frac{\sum_s f_s - \beta_0}{\beta_1} - \bar{W})^2) = \ln(\nu) + \frac{1-\rho}{\rho}\ln(\sum_{s=1}^{s=J} a_s f_s^{\rho}) + \ln(a_j f_j^{\rho-1} - a_1 f_1^{\rho-1}) \quad \text{for} \quad j = 2,...,J \tag{25}$$

Differentiate with respect to p_k:

$$\forall k = 1, ..., J, \quad \forall j = 2, ..., J$$
$$\frac{d}{dp_k}(\ln(p_j - p_1)) = \frac{2\kappa(W - \bar{W})}{\beta_1(1 + \kappa(W - \bar{W})^2)} \sum_s \frac{\partial f_s}{\partial p_k} + \frac{1-\rho}{\sum_s a_s f_s^\rho} \sum_s a_s f_s^{\rho-1} \frac{\partial f_s}{\partial p_k}$$
$$- (1-\rho) \frac{a_j f_j^{\rho-2} \frac{\partial f_j}{\partial p_k} - a_1 f_1^{\rho-2} \frac{\partial f_1}{\partial p_k}}{a_j f_j^{\rho-1} - a_1 f_1^{\rho-1}} \tag{26}$$

Regroup the terms $\frac{\partial f_j}{\partial p_k}$:

$$\forall k = 1, ..., J, \quad \forall j = 2, ..., J$$
$$\frac{d}{dp_k}(\ln(p_j - p_1)) = \sum_{s=1}^{s=J} \frac{\partial f_s}{\partial p_k} \Big(\frac{2\kappa(W - \bar{W})}{\beta_1(1 + \kappa(W - \bar{W})^2)} + (1-\rho) \frac{a_s f_s^{\rho-1}}{\sum_r a_r f_r^\rho} \Big)$$
$$+ (1-\rho) \frac{a_1 f_1^{\rho-2}}{a_j f_j^{\rho-1} - a_1 f_1^{\rho-1}} \frac{\partial f_1}{\partial p_k} - (1-\rho) \frac{a_j f_j^{\rho-2}}{a_j f_j^{\rho-1} - a_1 f_1^{\rho-1}} \frac{\partial f_j}{\partial p_k} \tag{27}$$

Define $\epsilon_{jk}^P = \frac{p_k}{f_j} \frac{\partial f_j}{\partial p_k}$ for $j \in \{1,...,J\}$ and $k \in \{1,...,J\}$. The previous expression becomes:

$$\forall k = 1, \ldots, J, \quad \forall j = 2, \ldots, J$$
$$p_k \frac{d}{dp_k}(\ln(p_j - p_1)) = \sum_{s=1}^{s=J} \epsilon_{sk}^P \Big(\frac{f_s}{\beta_1} \frac{2\kappa(W - \bar{W})}{(1 + \kappa(W - \bar{W})^2)} + \frac{1-\rho}{\sum_r \frac{a_r}{a_s} (\frac{f_r}{f_s})^\rho} \Big)$$
$$+ (1-\rho) \Big(\frac{\epsilon_{1k}^P}{\frac{a_j}{a_1} (\frac{f_j}{f_1})^{\rho-1} - 1} + \frac{\epsilon_{jk}^P}{\frac{a_1}{a_j} (\frac{f_1}{f_j})^{\rho-1} - 1} \Big) \tag{28}$$

Calibration of parameter ρ

Note that when $W = \bar{W}$, an assumption of Proposition 1, and for the special case $k = 1$ and $j = 2$, equation (38) becomes:

$$-\frac{p_1}{p_2 - p_1} = (1-\rho) \Big(\sum_{s=1}^{s=J} \frac{\epsilon_{s1}^P}{\sum_r \frac{a_r}{a_s} (\frac{f_r}{f_s})^\rho} + \frac{\epsilon_{11}^P}{\frac{a_2}{a_1} (\frac{f_2}{f_1})^{\rho-1} - 1} + \frac{\epsilon_{21}^P}{\frac{a_1}{a_2} (\frac{f_1}{f_2})^{\rho-1} - 1} \Big) \tag{29}$$

Note also that when $W = \bar{W}$, the first-order conditions in equations (15) and (16) become:

$$p_j - p_1 = \nu (a_j f_j^{\rho-1} - a_1 f_1^{\rho-1}) (\sum_j a_j f_j^\rho)^{\frac{1-\rho}{\rho}} \tag{30}$$

$$p_1 = \nu a_1 f_1^{\rho-1} (\sum_j a_j f_j^\rho)^{\frac{1-\rho}{\rho}} \tag{31}$$

Now divide the left- and right-hand sides of equations (30) and (31), we get:

$$\pi_j = \frac{a_j}{a_1} (\frac{f_j}{f_1})^{\rho-1} \qquad \text{for} \qquad j = 2,...,J \tag{32}$$

where $\pi_j = \frac{p_j}{p_1}$.

More generally, we can write the previous expression as:

$$\frac{\pi_r}{\pi_s} = \frac{a_r}{a_s} (\frac{f_r}{f_s})^{\rho-1} \qquad \text{for all } (r,s) \tag{33}$$

We rewrite equation (29) using relationship (33):

$$\frac{1}{1-\pi_2} = (1-\rho)\Big(\sum_{s=1}^{s=J} \frac{\epsilon_{s1}^P}{\sum_r \frac{\pi_r}{\pi_s}\frac{f_r}{f_s}} + \frac{\epsilon_{11}^P}{\pi_2 - 1} + \frac{\epsilon_{21}^P}{\frac{1}{\pi_2} - 1}\Big) \tag{34}$$

Using the fact that $\frac{f_r}{f_s} = \frac{\theta_r}{\theta_s}$, the previous relation becomes:

$$\frac{1}{1-\pi_2} = (1-\rho)\Big(\frac{\sum_{s=1}^{s=J} \epsilon_{s1}^P \pi_s \theta_s}{\sum_r \pi_r \theta_r} + \frac{\epsilon_{11}^P}{\pi_2 - 1} + \frac{\epsilon_{21}^P}{\frac{1}{\pi_2} - 1}\Big) \tag{35}$$

We can now solve for the parameter ρ:

$$\rho = 1 - \frac{1}{1-\pi_2} \times \frac{1}{\frac{\sum_{s=1}^{s=J} \epsilon_{s1}^P \pi_s \theta_s}{\sum_r \pi_r \theta_r} + \frac{\epsilon_{11}^P}{\pi_2 - 1} + \frac{\epsilon_{21}^P}{\frac{1}{\pi_2} - 1}} \tag{36}$$

The previous expression for ρ is equation (17).

Calibration of parameter $\{a_j\}$

Using the fact that $\frac{f_j}{f_1} = \frac{\theta_j}{\theta_1}$, equation (42) can be rewritten as:

$$\pi_j = \frac{a_j}{a_1}\left(\frac{\theta_j}{\theta_1}\right)^{\rho-1} \quad \text{for} \quad j = 2,\ldots,J \tag{37}$$

Summing over j and using the fact that $^{P}a_j = 1$, we get:

$$\sum_j \pi_j \left(\frac{\theta_j}{\theta_1}\right)^{1-\rho} = \frac{1}{a_1} \tag{38}$$

We thus obtain the following expression for a_1:

$$a_1 = \frac{\theta_1^{1-\rho}}{\sum_j \pi_j \theta_j^{1-\rho}} \tag{39}$$

Remaining coefficients $\{a_j\}$ are given by:

$$a_j = a_1 \pi_j \left(\frac{\theta_j}{\theta_1}\right)^{1-\rho} \quad \text{for} \quad j = 2,\ldots,J \tag{40}$$

6.3.4 Calibration of parameter v

When $W = \bar{W}$ we showed the following in equation (31):

$$p_1 = \nu a_1 f_1^{\rho-1} \Big(\sum_j a_j f_j^{\rho}\Big)^{\frac{1-\rho}{\rho}}$$

We can write the previous expression as follows:

$$\begin{aligned} p_1 &= \nu a_1^{\frac{1}{\rho}} \Big(\sum_j \frac{a_j}{a_1}\Big(\frac{f_j}{f_1}\Big)^{\rho}\Big)^{\frac{1-\rho}{\rho}} \\ &= \nu a_1^{\frac{1}{\rho}} \Big(\sum_j \frac{\pi_s \theta_s}{\theta_1}\Big)^{\frac{1-\rho}{\rho}} \\ &= \nu a_1^{\frac{1}{\rho}} \theta_1^{-\frac{1-\rho}{\rho}} \Big(\sum_j \pi_s \theta_s\Big)^{\frac{1-\rho}{\rho}} \end{aligned} \tag{41}$$

We can solve for v from the previous expression:

$$\nu = \frac{p_1 \theta_1^{\frac{1-\rho}{\rho}}}{a_1^{\frac{1}{\rho}} \left(\sum_j \pi_s \theta_s\right)^{\frac{1-\rho}{\rho}}} \tag{42}$$

6.3.5 Calibration of parameter κ

We rewrite first-order equation in equation (16) as:

$$\frac{p_1(1+\kappa(\frac{\sum_j f_j - \beta_0}{\beta_1} - \bar{W})^2) + 2\delta\zeta\kappa(I - \sum_j p_j f_j)(\frac{\sum_j f_j - \beta_0}{\beta_1} - \bar{W})}{(1+\kappa(\frac{\sum_j f_j - \beta_0}{\beta_1} - \bar{W})^2)^2} = \nu\eta f_1^{\rho-1}(\sum_{j=1}^{j=J} a_j f_j^{\rho})^{\frac{1-\rho}{\rho}} \tag{43}$$

Take logarithm:

$$\begin{aligned}
&\ln(p_1(1+\kappa(\frac{\sum_j f_j - \beta_0}{\beta_1} - \bar{W})^2) + 2\delta\zeta\kappa(I - \sum_j p_j f_j)(\frac{\sum_j f_j - \beta_0}{\beta_1} - \bar{W})) \\
&- 2\ln((1+\kappa(\frac{\sum_j f_j - \beta_0}{\beta_1} - \bar{W})^2)) \\
&= \ln(\nu) + \ln(\eta) - (1-\rho)\ln(f_1) + \frac{1-\rho}{\rho}\ln(\sum_{j=1}^{j=J} a_j f_j^{\rho})
\end{aligned} \tag{44}$$

Differentiate with respect to p_k:

$$\begin{aligned}
&\forall k = 1, ..., J \\
&\frac{\mathbb{1}_{k=1}(1+\kappa(W-\bar{W})^2) + \frac{p_1}{\beta_1} 2\kappa(W-\bar{W})\sum_s \frac{\partial f_s}{\partial p_k}}{p_1(1+\kappa(W-\bar{W})^2) + 2\delta\zeta\kappa(I - \sum_s p_s f_s)(W-\bar{W})} \\
&- \frac{2\delta\zeta\kappa}{\beta_1}\frac{\beta_1(f_k + \sum_s p_s \frac{\partial f_s}{\partial p_k})(W-\bar{W}) - (I - \sum_s p_s f_s)\sum_s \frac{\partial f_s}{\partial p_k}}{p_1(1+\kappa(W-\bar{W})^2) + 2\delta\zeta\kappa(I - \sum_s p_s f_s)(W-\bar{W})} \\
&- \frac{4}{\beta_1}\frac{\kappa(W-\bar{W})\sum_s \frac{\partial f_s}{\partial p_k}}{1+\kappa(W-\bar{W})^2} = (1-\rho)\frac{\sum_{s=1}^{s=J} a_s f_s^{\rho-1}\frac{\partial f_s}{\partial p_k}}{\sum_{s=1}^{s=J} a_s f_s^{\rho}} - \frac{1-\rho}{f_1}\frac{\partial f_1}{\partial p_k}
\end{aligned} \tag{45}$$

Regroup terms in $\frac{\partial f_j}{\partial p_k}$:

$$\begin{aligned}
&\forall k = 1, ..., J \\
&\frac{\mathbb{1}_{k=1}(1+\kappa(W-\bar{W})^2) + 2\delta\zeta\kappa f_k(W-\bar{W})}{p_1(1+\kappa(W-\bar{W})^2) + 2\delta\zeta\kappa(I - \sum_s p_s f_s)(W-\bar{W})} = \\
&\sum_{s=1}^{s=J}\frac{\partial f_s}{\partial p_k}\Big(-\frac{2\kappa}{\beta_1}\frac{p_1(W-\bar{W}) - \delta\zeta(\beta_1 p_s(W-\bar{W}) - (I - \sum_s p_s f_s))}{p_1(1+\kappa(W-\bar{W})^2) + 2\delta\zeta\kappa(I - \sum_s p_s f_s)(W-\bar{W})} \\
&+ \frac{4}{\beta_1}\frac{\kappa(W-\bar{W})}{1+\kappa(W-\bar{W})^2} + (1-\rho)\frac{a_s f_s^{\rho-1}}{\sum_{r=1}^{r=J} a_r f_r^{\rho}}\Big) - \frac{1-\rho}{f_1}\frac{\partial f_1}{\partial p_k}
\end{aligned} \tag{46}$$

Rewriting in terms of elasticity yields:

$$\forall k = 1, ..., J$$
$$p_k \frac{\mathbb{1}_{k=1}(1+\kappa(W-\bar{W})^2) + 2\delta\zeta\kappa p_k f_k (W-\bar{W})}{p_1(1+\kappa(W-\bar{W})^2) + 2\delta\zeta\kappa(I-\sum_s p_s f_s)(W-\bar{W})} =$$
$$\sum_{s=1}^{s=J} \epsilon_{f_s,p_k}\Big(-\frac{2\kappa f_s}{\beta_1}\frac{p_1(W-\bar{W}) - \delta\zeta(\beta_1 p_s(W-\bar{W}) - (I-\sum_s p_s f_s))}{p_1(1+\kappa(W-\bar{W})^2) + 2\delta\zeta\kappa(I-\sum_s p_s f_s)(W-\bar{W})}$$
$$+ \frac{4f_s}{\beta_1}\frac{\kappa(W-\bar{W})}{1+\kappa(W-\bar{W})^2} + \frac{1-\rho}{\sum_{r=1}^{r=J}\frac{a_r}{a_s}(\frac{f_r}{f_s})^\rho}\Big) - (1-\rho)\epsilon_{f_1,p_k} \tag{47}$$

We write the previous expression for $k = 1$ and when $W = \bar{W}$:

$$1 = \sum_{s=1}^{s=J} \epsilon_{s1}^P \Big(-\frac{2\kappa\delta\zeta f_s}{\beta_1 p_1}(I - \sum_s p_s f_s) + \frac{1-\rho}{\sum_{r=1}^{r=J}\frac{a_r}{a_s}(\frac{f_r}{f_s})^\rho}\Big) - (1-\rho)\epsilon_{11}^P \tag{48}$$

We rearrange the previous equation slightly as follows:

$$1 = -\frac{2\kappa\delta\zeta}{\beta_1 p_1}(I - \sum_s p_s f_s)\sum_{s=1}^{s=J}\epsilon_{s1}^P f_s + (1-\rho)\Big(\frac{\sum_{s=1}^{s=J}\epsilon_{s1}^P \pi_s \theta_s}{\sum_{r=1}^{r=J}\pi_r\theta_r}\Big) - (1-\rho)\epsilon_{11}^P \tag{49}$$

From the previous equation, we can solve for κ:

$$\kappa = \frac{\beta_1 p_1}{2\delta\zeta(I-\sum_s p_s f_s)\sum_{s=1}^{s=J}\epsilon_{s1}^P f_s}\Big(-1 + (1-\rho)p_1(\frac{\sum_{s=1}^{s=J}\epsilon_{s1}^P\pi_s\theta_s}{\sum_{r=1}^{r=J}\pi_r\theta_r} - \epsilon_{11}^P)\Big) \tag{50}$$

We can write the previous expression as follows:

$$\kappa = -\frac{\beta_1 p_1}{2\delta\zeta(\beta_0+\beta_1\bar{W})\sum_{j=1}^{j=J}\epsilon_{j1}^P\theta_j}\frac{1 - p_1(1-\rho)(\frac{\sum_{j=1}^{j=J}\epsilon_{j1}^P\pi_j\theta_j}{\sum_{r=1}^{r=J}\pi_r\theta_r} - \epsilon_{11}^P)}{I - p_1(\beta_0+\beta_1\bar{W})\sum_{j=1}^{j=J}\pi_j\theta_j} \tag{51}$$

The previous expression is equation (20).

B. Proof of Proposition 2

Proposition 2 states that there exist two J-by-J matrices Γ and Λ so that $\Gamma\epsilon^P = \Lambda$. Let γ_{jk} and λ_{jk} denote the coefficients of Γ and Λ, respectively. We have:

$$\sum_{s=1}^{s=J}\gamma_{js}\epsilon_{sk}^P = \lambda_{jk} \quad \forall(j,k) \in J^2$$

We show to write equation (28) in matrix form first. Using equation (32) and when W= $\bar{W}$ equation (28) can be written as follows:

$$\forall k = 1, ..., J, \quad \forall j = 2, ..., J$$
$$p_k\frac{d}{dp_k}(\ln(p_j - p_1)) = (1-\rho)\Big(\frac{\sum_{s=1}^{s=J}\epsilon_{sk}^P\pi_s\theta_s}{\sum_r\pi_r\theta_r} + \frac{\epsilon_{1k}^P}{\pi_j - 1} + \frac{\epsilon_{jk}^P}{\frac{1}{\pi_j}-1}\Big) \tag{52}$$

Now let us examine the left-hand side more carefully. When $k = 1$, we have:

$$p_1 \frac{d}{dp_1}(\ln(p_j - p_1)) = \frac{1}{1-\pi_j} \quad \forall j = 2, ..., J \tag{53}$$

Similarly, when $k = j$, we have:

$$p_j \frac{d}{dp_j}(\ln(p_j - p_1)) = \frac{1}{1-\frac{1}{\pi_j}} \quad \forall j = 2, ..., J \tag{54}$$

Finally, it is easy to show that $p_k \frac{d}{dp_k}(\ln(p_j - p_1)) = 0$ when $j = k$.

Let γ_{jk} and λ_{jk} be defined by:

$$\gamma_{jk} = (1-\rho)\left(\frac{\pi_k \theta_k}{\sum_{r=1}^{r=J} \pi_r \theta_r} + \frac{\mathbb{1}_{k=1}}{\pi_j - 1} + \frac{\mathbb{1}_{k=j}}{\frac{1}{\pi_j} - 1}\right) \quad \forall j = 2, ..., J \quad k = 1, ..., J$$

And

$$\lambda_{1k} = 0 \qquad \forall k = 2,...,J$$

$$\lambda_{j1} = \frac{1}{1-\pi_j} \qquad \text{and} \qquad \lambda_{jj} = \frac{1}{1-\frac{1}{\pi_j}} \quad \forall j = 2, ..., J \tag{55}$$

$$\lambda_{jk} = 0 \qquad \text{for all } j > 1, \qquad k > 1, \qquad \text{and } j = k$$

Then the system of first-order conditions in equation (52) can be written as:

$$\sum_{s=1}^{s=J} \gamma_{js} \epsilon_{sk}^P = \lambda_{jk} \quad \forall j = 2, ..., J \quad k = 1, ..., J \tag{56}$$

where the coefficients γ_{jk} and λ_{jk} are defined in equation (55).

To complete the remainder of the proof for Proposition 2, we need to derive an expression for γ_{1k} and λ_{1k} for all $k = 1,...,J$. We rewrite equation (49) as follows:

$$\begin{aligned} 1 = &-\frac{2\kappa\delta\zeta}{\beta_1}(\beta_0 + \beta_1 \bar{W})\Big(\frac{I}{p_1} - (\beta_0 + \beta_1 \bar{W}) \sum_{s=1}^{s=J} \pi_s \theta_s\Big) \sum_{s=1}^{s=J} \epsilon_{s1}^P \theta_s \\ &+ (1-\rho)\Big(\frac{\sum_{s=1}^{s=J} \epsilon_{s1}^P \pi_s \theta_s}{\sum_{r=1}^{r=J} \pi_r \theta_r} - \epsilon_{11}^P\Big) \end{aligned} \tag{57}$$

Let γ_{1k} and λ_{1k} be defined by:

$$\begin{aligned} \gamma_{1k} = &-\frac{2\kappa\delta\zeta\theta_k}{\beta_1}(\beta_0 + \beta_1 \bar{W})\Big(\frac{I}{p_1} - (\beta_0 + \beta_1 \bar{W}) \sum_{r=1}^{r=J} \pi_r \theta_r\Big) \\ &+ (1-\rho)\Big(\frac{\pi_k \theta_k}{\sum_{r=1}^{r=J} \pi_r \theta_r} - \mathbb{1}_{k=1}\Big) \quad \forall k = 1, ..., J \end{aligned} \tag{58}$$

and

$$\lambda_{11} = 1 \qquad \text{and} \qquad \lambda_{1k} = 0 \qquad \forall k = 2,...,J$$

Then the system of first-order conditions in equation (56) can be written as:

$$\sum_{s=1}^{s=J} \gamma_{1s} \epsilon_{sk}^{P} = \lambda_{1k} \quad \forall k = 1, ..., J \tag{59}$$

where the coefficients γ_{jk} and λ_{jk} are defined in equation (58). Together, equations (56) and (59) show that the first-order conditions can be written as $\Gamma\epsilon^{P} = \Lambda$.

To complete the end of the proof of Proposition 2, we assume that the matrix Γ is invertible from the left. In that case, the price and cross-price elasticity matrix is given by:

$$\epsilon^{P} = \Gamma^{-1}\Lambda$$

where the coefficients for Γ and Λ are defined in equations (55) and (58).

C. Proposition 3

Recall that the steady state weight is equal $W = \frac{\sum_j f_j - \beta_0}{\beta_1}$.

Taking partial derivatives of the previous expression with p_j, we have:

$$\epsilon_j^{W} = \frac{p_j}{W}\frac{\partial W}{\partial p_j} = \frac{p_j}{W\beta_1}\sum_{s=1}^{s=J}\frac{\partial f_s}{\partial p_j} \quad \forall j = 1, ..., J \tag{60}$$

We rewrite the previous expression as:

$$\epsilon_j^{W} = \frac{1}{W\beta_1}\sum_{s=1}^{s=J}\frac{p_j}{f_s}\frac{\partial f_s}{\partial p_j} f_s \quad \forall j = 1, ..., J \tag{61}$$

Since $\epsilon_{sj}^{P} = \frac{p_j}{f_s}\frac{\partial f_s}{\partial p_j}$ and $\theta_s = \frac{f_s}{\beta_0 + \beta_1 W}$, we have:

$$\epsilon_j^{W} = \frac{\beta_0 + \beta_1 W}{W\beta_1}\sum_{s=1}^{s=J}\epsilon_{sj}^{P}\theta_s \quad \forall j = 1, ..., J \tag{62}$$

Rewriting in matrix complete the proof of Proposition 3 with:

$$\epsilon^{W} = (1 + \frac{\beta_0}{W\beta_1})\epsilon^{P}\theta \tag{63}$$

The price and cross-price elasticity for weight for men and women are shown in Table 6.